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# **Comment on “ Earthquake cycles and physical modeling of the process leading up to a large earthquake “**

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The modeling of earthquake initiation and subsequent rupture propagation requires the use of a fault constitutive law controlling the traction evolution and the slip acceleration that yields a finite energy flux at the crack tip. The determination of the temporal evolution of dynamic traction during the propagation of an earthquake rupture has been the major task of many recent investigations. Ohnaka (2004) presented a detailed discussion in favor of a “*rational*” governing law for earthquake ruptures which is consistent with laboratory experiments and, according to the author, is based on the “physics of rock friction and fracture”. He concluded that this constitutive law must be slip-dependent, because the slip dependency is a “more fundamental property of the shear rupture than the rate-dependency”. Ohnaka (2004) suggested this slip-dependent law also to model the whole seismic cycle (therefore including earthquake nucleation and long-term fault re-strengthening) and considered the proposed law as a governing relation unifying the different features of the earthquake failure process.

In order to support the proposed slip-dependent law, Ohnaka (2004) presented a questionable comparison between the slip- and the rate- and state- dependent (RS hereinafter) constitutive laws (see section 2 of that paper). As previously mentioned, his discussion has been focused on demonstrating that the slip dependency is more important than the rate dependency. Our goal here is not to open a debate on the more appropriate constitutive law neither to support RS friction against slip-dependent laws. Conversely, we aim to provide an alternative interpretation of the slip dependency of shear traction as well as to discuss the reasons why RS friction laws yield a traction evolution consistent with slip-weakening (SW in the following). Although we share the desirable perspective to find a unified constitutive law to model the earthquake initiation, propagation and arrest, we think that the complexity and the diversity of real fault zone structures make this task difficult to be reached. We will try to explain this issue in the following.

It is generally accepted that the dynamic fault weakening during the rupture propagation can be represented by a traction evolution showing a dominant SW behavior (Figure 1). This evolution is

characterized by a traction increase to the upper yield stress ( $t_u$ ), which is followed by a decrease to a kinetic friction level ( $t_p$ ) during a time interval that defines the duration of the breakdown process. The amount of slip required to the traction drop from the peak stress to the kinetic friction level is named the characteristic slip-weakening distance ( $d_0$ ). The cohesive zone represents the region of shear stress degradation near the tip of a propagating rupture (Ida, 1972; Andrews, 1976a, 1976b; Ohnaka and Yamashita, 1989). Slip-weakening has been observed in laboratory experiments (Okubo and Dieterich, 1984; Ohnaka et al., 1987), it has been proposed in theoretical studies (Campillo and Ionescu, 1997; Ohnaka, 2003 and references therein) and it has been used in numerical simulations of earthquake ruptures (Day, 1982; Olsen et al., 1997, among many others). The SW behavior of shear traction evolution has been demonstrated to occur also when RS friction laws are adopted (see Bizzarri and Cocco, 2003 and references therein). Therefore, there exists a general consensus that SW characterizes dynamic fault weakening. It is also accepted that the shape of the traction versus slip curve can be non-linear (i.e.,  $\partial t / \partial u \neq \text{constant}$ , where  $t$  is the shear traction and  $u$  is the fault slip; see Abercrombie and Rice, 2005) and may exhibit a slip-hardening phase preceding the slip-weakening phase (i.e.,  $t$  increases with  $u$  and reaches its peak value in correspondence of a fault slip value  $D_a$ ; see for instance Ohnaka and Yamashita, 1989). For this reason, in many numerical simulations of dynamic rupture propagation, a SW constitutive law (Andrews, 1976a, 1976b) is assumed in order to prescribe the traction evolution within the cohesive zone and to model the propagation of seismic waves in the medium surrounding the fault. However, there still exist many open questions concerning the physical processes affecting the dynamic traction evolution and therefore controlling the SW behavior. Related to this issue, there is also the problem of choosing the best analytical formulation to appropriately model the constitutive behavior as well as to decide the independent physical observables adopted in the proposed law (see for instance equation (2) in Bizzarri and Cocco, 2005a).

Ohnaka (2004) raised serious concerns and criticism on the use of RS friction laws made by Bizzarri et al. (2001) and Cocco and Bizzarri (2002). He considered “logically” inconsistent one of the conclusions of these authors who stated that, in order to have SW with RS dependent friction laws, there is no need to assume that friction is independent of slip rate. We believe that there are several shortcomings in the Ohnaka’s discussion of this issue and we wish to better explain the way in which RS friction laws can yield a traction evolution showing SW during the spontaneous rupture propagation. To this goal we remind here the constitutive relations characterizing the slowness law used by Cocco and Bizzarri (2002) and Bizzarri and Cocco (2003), which is defined by two equations:

$$\begin{aligned} \tau &= \left[ \mathbf{m}_* - a \ln \left( \frac{v_*}{v} + 1 \right) + b \ln \left( \frac{\mathbf{Y} v_*}{L} + 1 \right) \right] \mathbf{s}_n^{eff} \\ \frac{d}{dt} \mathbf{Y} &= 1 - \frac{\mathbf{Y} v}{L} \end{aligned} \quad (1)$$

the former is often named the governing relation, while the latter is called the evolution equation. In (1)  $v$  is the slip velocity,  $\mathbf{Y}$  is the state variable,  $\mathbf{m}_*$  and  $v_*$  are reference values for the friction coefficient and for the slip velocity, respectively,  $a$ ,  $b$  and  $L$  are the three constitutive parameters and  $\mathbf{s}_n^{eff}$  is the effective normal stress. Dieterich and Kilgore (1996) have pointed out that, if the slip velocity is constant and  $\mathbf{Y} v/L \gg 1$ , the state variable evolution can be written as  $\mathbf{Y} = \mathbf{Y}_0 \exp(-u/L)$  ( $\mathbf{Y}_0$  being the initial value of the state variable) and therefore the slip dependence of total traction can be derived analytically. However, the assumption of a constant slip rate is not applicable to the dynamic traction evolution within the cohesive zone: it is well known that slip velocity varies over several order of magnitudes during the breakdown time (see Figure 1). According to the results of their numerical simulations, Cocco and Bizzarri (2002) demonstrated that RS dependent friction laws yield a dynamic traction evolution characterized by SW, with slip velocity being not constant and varying up to several m/s. Thus, they concluded that the assumption of a constant slip velocity is not necessary to have a SW behavior of dynamic traction. We believe that this conclusion is correct. Ohnaka (2004) considers this issue “logically inconsistent” because of the effect of the high velocity cutoff incorporated in equation (1) through the presence of the +1 term in the arguments of the logarithms. First of all, it has to be emphasized that, even if the first logarithmic term in (1) would saturate at high slip rates, the traction dependence on slip rate would remain in the governing equation, because also the state variable depends on the fault slip velocity. Therefore, we underline that the slip velocity dependence of the shear traction is contained in both the logarithmic terms and not only in the first (usually named the “direct effect of friction”). This is clearly demonstrated by the simulations presented in Tinti et al. (2004), who have shown that, also using different high slip velocity cutoff for the direct effect of friction, the SW behavior is always retrieved. Moreover, we have verified that the effect of the +1 terms does not affect the traction evolution when  $v_*$  is greater than 0.1 m/s (this condition was adopted in the simulations shown by Cocco and Bizzarri, 2002). For values of  $v_*$  smaller than 0.1 m/s the +1 term affects the direct effect of friction, but also in this case the fault shear traction still displays an evident SW behavior driven by the state variable evolution (see Figure 1 and Bizzarri and Cocco, 2003, for the details).

We agree with Ohnaka (2004) that the  $v_*$  term cannot have an arbitrary value and has to be constrained by observations. In the literature, the only direct experimental measure of parameter  $v_*$  is in Okubo and Dieterich (1986); these authors performed laboratory experiments and interpreted their observations using the RS formulation. They estimated (see Table 1 in Okubo and Dieterich, 1986) the parameter  $c_3$ , that is equal to  $v_*/L$  in our notation. It is important to remark that, accordingly to these experimental results, the value of  $v_*$  depends on the scale at which the numerical experiment is performed: at the laboratory scale (fault size of the order of meters and  $L \sim 1 \mu\text{m}$ )  $v_* \sim 0.0001 \text{ m/s}$ , while for a real-world fault (size of the order of kilometers and  $L \sim 1 \text{ cm}$ , Scholz, 1998)  $v_* \sim 0.15 \text{ m/s}$ . Bizzarri and Cocco (2003) demonstrated that the scaling between  $d_0^{eq}$  and  $L$  depends on the initial configuration of the sliding surface. This is also consistent with the Ohnaka (2003, 2004) conclusions that the characteristic SW distance and fracture energy are scale-dependent quantities. Therefore, we believe that the simulations presented by Cocco and Bizzarri (2002) and Bizzarri and Cocco (2003) demonstrate that RS friction laws yield a traction evolution within the cohesive zone consistent with the SW behavior. To this goal we wish to emphasize the similarity between the traction evolution with slip inferred in numerical simulations governed by RS dependent friction laws (see Figure 1) and the experimental results displayed by Ohnaka and Yamashita (1989) in Figure 2 of their paper. In our opinion, this represents a further validation of the agreement between numerical simulations and observational evidence.

According to the results of laboratory and numerical experiments, we agree with Ohnaka that the traction evolution within the cohesive zone during the dynamic rupture propagation must show a SW behavior. Nevertheless, we disagree with him on the statement that this is the “unified” constitutive law that must be slip-dependent. Different constitutive relations allow the simulation of a traction evolution consistent with SW, each of which having different independent physical observables to analytically represent shear traction. We have previously discussed slip-dependent and RS dependent friction laws, but other mechanisms can affect the constitutive process and the dynamic traction evolution that might be described by different physical parameters (see Sleep, 1997, for instance). In other words, we believe that there are many competing physical mechanisms (such as mechanical lubrication or thermal pressurization) controlling the constitutive process, which are not necessarily described by a slip-dependent law and that can yield a dynamic traction evolution characterized by SW. These distinct physical mechanisms can be adopted to describe tectonic faults because of the diversity of real fault zones.

Recent observations of active faults suggest that the fault zone is composed by a damage zone containing a narrow fault core where a relatively thin slipping zone (mm ÷ cm thickness) is located

(Chester and Chester, 1998; Sibson, 2003 and references therein). In that region different physical mechanisms may occur: intact rock fracturing, comminution and pulverization (Wilson et al., 2004), subsequent gouge creation, gouge fragmentation (Sleep et al., 2000), frictional heating and melting as well as chemical environment changes. Moreover, the seismogenic zone may contain fluids that can migrate and affect the fault weakening, through hydrodynamic lubrication (Brodsky and Kanamori, 2001) and thermal pressurization (Andrews, 2002; Cocco and Bizzarri, 2004; Bizzarri and Cocco, 2005b, 2005c). We would also emphasize that the behavior of the damage zone (anelastic deformations, permeability and porosity evolutions) may influence the fault friction. All of these phenomena should be considered in the formulation of a unifying constitutive model that describes the evolution of the slipping surface. It is likely to expect that different configurations of fault zones will be described by different constitutive relations, which have been indeed proposed in the literature. For instance, the evolution of gouge materials has been simulated in laboratory (Mair and Marone, 2000; Richardson and Marone, 1999) and in numerical experiments (Sleep et al., 2000). Therefore, within this variety of possible configurations of a fault zone, we do not believe that it is justified to state that the constitutive law representing these processes must be slip-dependent and that this slip dependency is associated to the “correct representation of the physics of friction and fracture”.

Recent results from laboratory experiments further corroborate that a slip-dependent constitutive law cannot account for the distinct physical processes characterizing faulting and dynamic rupture episodes during earthquakes. Di Toro et al. (2004) have shown an extraordinary progressive decrease in frictional resistance with increasing slip velocity: these experimental results suggest that the friction coefficient (that is, at constant normal stress, the shear traction) depends both on slip and slip velocity. Tullis and Goldsby (2003) have modeled these data and interpreted the friction reduction at high slip rates in terms of flash heating of contact asperities (Rice, 1999). Mair and Marone (2000) performed laboratory experiments to study friction of simulated fault gouge at relatively high normal stress (25 to 70 MPa) and they found a dramatic reduction in frictional strength with accumulated slip. They interpreted their laboratory results in terms of a slip-dependent textural evolution of gouge and found an evident velocity-weakening behavior as well as negligible shear heating. Thus, many observations and laboratory experiments suggest that fault gouge and the properties of contact surface evolve during slip episodes; this evolution depends on the slip, slip rate, normal stress as well as the degree of strain localization. Our opinion is that the interpretation of these observations does not allow us to distinguish the independent variables controlling the dynamic fault weakening.

It is important to point out that the different physical mechanisms described above represent either microscopic or macroscopic processes. Melting caused by frictional heating at high slip rates (Tsutsumi and Shimamoto, 1997) represents a further macroscopic frictional behavior of a fault zone (see also Brodsky and Kanamori, 2001). Ohnaka (2003) emphasizes that the shear traction and the slip appearing in the constitutive formulation have to be regarded as macroscopic quantities (i.e., defined as macroscopic averages) describing the shear stress acting on both walls of the fault zone thickness as well as their relative displacement. In this context, the shear stress adopted in the constitutive law and used to characterize the macroscopic dynamic fault weakening is not the shear stress acting on a individual asperity of contact surface, micro-crack or gouge fragment within the slipping zone. We fully agree with this view of the earthquake failure process: at the present state of knowledge we cannot expect to be able to model the details and the complexity of the inhomogeneous processes occurring in the localized zone where the strain is concentrated. For the same reasons we believe that a simple slip-dependent law cannot be proposed to characterize the traction evolution. Further investigations are needed to answer this problem quantitatively.

Following the discussion presented above, we would like to propose an alternative interpretation of the shear traction dependence on slip (i.e., the SW behavior). Our opinion is that, if we restrict our goal to find an analytical expression capable to represent the traction evolution within the cohesive zone during the dynamic earthquake propagation, thus a slip-dependent law is a candidate solution. In this case, “unifying” slip-dependent law means that we only reproduce the traction evolution independently of the physical mechanisms controlling the constitutive process. In this context, we agree with the Ohnaka’s conclusions. Conversely, we believe that the goal of representing the whole variety of constitutive processes and the dynamic fault weakening during earthquake failures with a slip-dependent law might be misleading for future developments of research in this field.

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## Figure Captions

**Figure 1.** Results of numerical experiments of a 2-D pure in-plane fully dynamic spontaneous rupture on a planar fault obeying to equation (1). The numerical method is the Finite Difference code described in Bizzarri et al. (2001) and the constitutive parameters are those cited in the caption of Figure 1 of Cocco and Bizzarri (2002). Different simulations are performed changing only the value of the reference fault slip velocity  $v_*$  from 1 mm/s up to 100 m/s and consequently of the initial pre-stress, that is assumed to be the steady state (see Bizzarri et al., 2001 for further numerical details). (a) Traction vs. slip (i.e., slip-weakening curve); (b) Traction vs. slip velocity (i.e., phase portrait). The two solid lines indicate the curves corresponding to lab-values, while dashed curves are computed for larger values of  $v_*$ .